August 1995

GSI-Preprint-95-46
hep-ph/9508318

Photon Intensity Interferometry for Expanding Sources

Leonid V. Razumov*, Hans Feldmeier§

Gesellschaft für Schwerionenforschung, Postfach 110552,
D-64220 Darmstadt, Germany

Abstract

Using Quantum Field Theory we derive a general formula for the double inclusive spectra of photons radiated by a system in local equilibrium. The derived expression differs significantly from the one mostly used up to now in photon intensity interferometry of heavy–ion collisions. We present a covariant expression for double inclusive spectra adapted for usage in numerical simulations. Application to a schematic model with a Bjørken type expansion gives strong evidence for the need of reinvestigating photon–photon correlations for expanding sources.

Submitted to Physics Letters B.

*E.Mail: razumov@clri6a.gsi.de (URL⇒ http://www.gsi.de/groups/the/razumov.html)
§E.Mail: h.feldmeier@gsi.de

Double inclusive spectra of hard photons radiated at the early stage of heavy–ion collisions have become an important subject of theoretical [1, 2, 3] and experimental [4] investigations. Since these photons originate from very energetic collisions and interact very little with the matter they are traversing, they may carry signatures of the hot and dense phase of the matter, in particular of the quark–gluon plasma.

Except for the work of D. Neuhauser [1] in which he treated a static source, usually one is analyzing the correlations in two–photon coincidence measurements with expressions based on the corresponding ones for scalar bosons [5] augmented with a degeneracy factor which is supposed to take care of the spin structure of the photons.

The aim of this work is to derive in a quantum–field treatment covariant expressions for the two–photon density and herewith the corresponding correlation function. In particular we pay attention to charge conservation and to the fact that massless photons have only two helicity eigenstates. Due to these properties averaging over all spin directions results in a reduction factor of one–half compared to the scalar case only if the momenta of the two photons are parallel. For a finite angle, however, the interference is reduced, so that the deduced size of the source is smaller than the one obtained from the correlation function for scalar bosons. For Bjørken type hydrodynamics we estimate a difference in the apparent source size of about 30% in rapidity. Therefore, we conclude that the "pion–inspired" formulas for the Bose–Einstein correlation function is not applicable for photon intensity interferometry studies, especially if one deals with an expanding source.

The finally measured number of photons per invariant momentum bin with polarization λ is given by the density matrix of the final state ρ_f which can be expressed in terms of the initial density matrix ρ_i and the S–matrix using the relation $\rho_f = S\rho_i S^\dagger$.

$$P_1^\lambda(\boldsymbol{k}) \;\equiv\; k^0\frac{dN^\lambda}{d^3\boldsymbol{k}} = Tr\{\rho_f c_\lambda^\dagger(\boldsymbol{k})c_\lambda(\boldsymbol{k})\} = Tr\{\rho_i S^\dagger c_\lambda^\dagger(\boldsymbol{k})c_\lambda(\boldsymbol{k})S\} \tag{1}$$

The same consideration as above leads to the expression for the number of photon pairs

$$P_2^{\lambda_1\,\lambda_2}(\boldsymbol{k}_1,\boldsymbol{k}_2) \equiv k_1^0 k_2^0\frac{d^2N^{\lambda_1\lambda_2}}{d^3\boldsymbol{k}_1 d^3\boldsymbol{k}_2} = Tr\{\rho_f c_{\lambda_2}^\dagger(\boldsymbol{k}_2)c_{\lambda_1}^\dagger(\boldsymbol{k}_1)c_{\lambda_1}(\boldsymbol{k}_1)c_{\lambda_2}(\boldsymbol{k}_2)\} \;=$$

$$= Tr\{\rho_i S^\dagger c_{\lambda_2}^\dagger(\boldsymbol{k}_2)c_{\lambda_1}^\dagger(\boldsymbol{k}_1)c_{\lambda_1}(\boldsymbol{k}_1)c_{\lambda_2}(\boldsymbol{k}_2)S\} \quad . \tag{2}$$

If we separate the interaction part of the Lagrangian into $L_s^{int}(x)$ for strong interactions and $J^\mu(x)A_\mu(x)$ for the coupling to the electromagnetic field the S–matix can be written in the interaction picture by the time–ordered exponential [6]

$$S \;=\; \mathcal{T}\exp\left\{i\int d^4x\Big(L_s^{int}(x) + J^\mu(x)A_\mu(x)\Big)\right\} \quad . \tag{3}$$

To proceed further we assume [7] that the initial state does not contain hard photons, which means

$$c_\lambda(\boldsymbol{k})\rho_i \;=\; \rho_i c_\lambda^\dagger(\boldsymbol{k}) \;=\; 0 \quad . \tag{4}$$

The condition (4) together with the identities

$$\Big[c_\lambda(\boldsymbol{k})\,,\,S\Big] = i\mathcal{T}\Big(J_\lambda(k)S\Big) \quad , \tag{5}$$

$$\Big[S^\dagger\,,\,c_\lambda^\dagger(\boldsymbol{k})\Big] = -i\tilde{\mathcal{T}}\Big(S^\dagger J_\lambda^{\,\dagger}(k)\Big) \quad ,$$

where $k^\mu = (|\boldsymbol{k}|, \boldsymbol{k})$ and $J_\lambda(k)$ is the Fourier transform of the transverse current operator given by

$$J_\lambda(k) \equiv \overset{*}{\epsilon}{}_\mu^\lambda(k)\int d^4x\, \mathrm{e}^{ixk} J^\mu(x) \tag{6}$$

$$J_\lambda^{\,\dagger}(k) \equiv \epsilon_\mu^\lambda(k)\int d^4x\, \mathrm{e}^{-ixk} J^\mu(x) \quad ,$$

allow us to write one– and two–body densities via the chronological ($\mathcal{T}$) and antichronological ($\tilde{\mathcal{T}}$) products of the current operators:

$$P_1^\lambda(\boldsymbol{k}) = Tr\Big\{\rho_i\tilde{\mathcal{T}}\Big(S^\dagger J_\lambda^{\,\dagger}(k)\Big)\mathcal{T}\Big(J_\lambda(k)S\Big)\Big\} \tag{7}$$

$$P_2^{\lambda_1\lambda_2}(\boldsymbol{k}_1,\boldsymbol{k}_2) = Tr\Big\{\rho_i\tilde{\mathcal{T}}\Big(S^\dagger J_{\lambda_2}^{\,\dagger}(k_2)J_{\lambda_1}^{\,\dagger}(k_1)\Big)\mathcal{T}\Big(J_{\lambda_1}(k_1)J_{\lambda_2}(k_2)S\Big)\Big\} \quad . \tag{8}$$

All currents in the expressions written above are operators in the *interaction* representation. Utilizing properties of time ordering and of the S–matrix we can write the one– and two–photon spectra in terms of the current operators in the *Heisenberg* representation ($\hat{J}$) [8] as

$$P_1^\lambda(\boldsymbol{k}) = Tr\Big\{\rho_i\hat{J}_\lambda^{\,\dagger}(k)\hat{J}_\lambda(k)\Big\} \tag{9}$$

$$P_2^{\lambda_1\lambda_2}(\boldsymbol{k}_1,\boldsymbol{k}_2) = Tr\Big\{\rho_i\tilde{\mathcal{T}}\Big(\hat{J}_{\lambda_2}^{\,\dagger}(k_2)\hat{J}_{\lambda_1}^{\,\dagger}(k_1)\Big)\mathcal{T}\Big(\hat{J}_{\lambda_1}(k_1)\hat{J}_{\lambda_2}(k_2)\Big)\Big\} \quad . \tag{10}$$

Up to now we have made no approximation to tackle the unsolvable many–body problem. Expressions (9) and (10) are still exact but they are recast into a form which is better suited for approximations than the original definitions (1) and (2). Formally they contain only operators of the strongly interacting system.

In the following we apply eqs. (9) and (10) to a highly excited system which due to short–ranged hard collisions of the charged constituents is radiating energetic photons. The collisions cause locally rapid changes in the electric charge current density $J^\mu(x)$,

which result in non–zero Fourier components in $J^\mu(k)$ for large $|\boldsymbol{k}|$. The energetic photons are weakly interacting so that, once they are created, they are assumed to leave the hadron system without further interactions. Therefore, for calculating the final photon distributions one needs a local production rate which is then integrated over space and time. Eq. (9) is of this type. In coordinate space it reads

$$P_1^\lambda(\boldsymbol{k}) = \overset{*}{\epsilon}{}^\lambda_{\mu_1}(k)\epsilon^\lambda_{\mu_2}(k)\int d^4\bar{x}\int d^4\Delta x\, \mathrm{e}^{-ik\Delta x}\, Tr\Big\{\rho_i \hat{J}^{\mu_1}(\bar{x}+\frac{\Delta x}{2})\hat{J}^{\mu_2}(\bar{x}-\frac{\Delta x}{2})\Big\} \qquad (11)$$

where the local rate is given by the integral over Δx of the current–current correlator.

The following approximations, which will lead to an expression for the correlator adapted for usage in numerical simulations for heavy–ion collisions, are based upon three physical assumptions:

(i) The hadronic correlations in the system are of short range in space–time. The typical correlation length is about the mean free path λ_s of the strongly interacting particles. Therefore,

$$< A(x)B(y) > \approx < A(x) >< B(y) > \qquad \text{if } \ |\boldsymbol{x}-\boldsymbol{y}| \gtrsim \lambda_s \ \text{ or } \ |x^0-y^0| \gtrsim \lambda_s \ .$$

(ii) The hadronic mean free path λ_s is much less than the characteristic size of variations of the macroscopic variables in space and time, denoted by L .

(iii) Only hard photons with high momenta $|\boldsymbol{k}|$ are considered ($|\boldsymbol{k}|\lambda_s \gtrsim 1$) , for which the slowly varying collective current $<\hat{J}^\mu(x)>$ (proportional to the collective 4–velocity field $u^\mu(x)$) does not contribute, i.e. $<\hat{J}^\mu(k)> \approx 0$ for high enough $|\boldsymbol{k}|$.

For instance, applying these assumptions to (11) means that the integral in Δx effectively spreads over the 4–volume of the order λ_s^4 and the single inclusive spectrum $P_1(\boldsymbol{k})$ is therefore proportional to $L^4\lambda_s^4$.

Inside the trace on the right–hand side of (11) the mean time $\bar{x}^0$ can be moved over to the statistical operator such that

$$Tr\Big\{\rho_i \hat{J}^{\mu_1}(\bar{x}+\frac{\Delta x}{2})\hat{J}^{\mu_2}(\bar{x}-\frac{\Delta x}{2})\Big\} = Tr\Big\{\rho(\bar{x}^0)\hat{J}^{\mu_1}(\tilde{x}+\frac{\Delta x}{2})\hat{J}^{\mu_2}(\tilde{x}-\frac{\Delta x}{2})\Big\} \ , \qquad (12)$$

where $\tilde{x} = (0, \boldsymbol{x})$ and $\rho(\bar{x}^0) = \mathrm{e}^{-iH\bar{x}^0}\rho_i \mathrm{e}^{iH\bar{x}^0}$ is the solution of the Liouville equation for the density matrix. For the case of local equilibrium, where the time–dependence enters the density matrix $\rho(\bar{x}^0)$ only through the thermodynamic quantities, like for example temperature $T(\bar{x}^0, \boldsymbol{x})$, 4–velocity $u^\mu(\bar{x}^0, \boldsymbol{x})$ etc., the identity (12) is a starting point to express the current–current correlator in terms of thermodynamic variables.

The double inclusive spectrum is given by the four–point correlator (10) and like any other Green function can be written in terms of connected parts as

$$P_2^{\lambda_1\lambda_2}(\boldsymbol{k}_1,\boldsymbol{k}_2) = P_1^{\lambda_1}(\boldsymbol{k}_1)P_1^{\lambda_2}(\boldsymbol{k}_2) + |<\hat{J}^\dagger_{\lambda_1}(k_1)\hat{J}_{\lambda_2}(k_2)>|^2 + |<\mathcal{T}\Big(\hat{J}_{\lambda_1}(k_1)\hat{J}_{\lambda_2}(k_2)\Big)>|^2$$

$$+ \ \ll\tilde{\mathcal{T}}\Big(\hat{J}^\dagger_{\lambda_2}(k_2)\hat{J}^\dagger_{\lambda_1}(k_1)\Big)\mathcal{T}\Big(\hat{J}_{\lambda_1}(k_1)\hat{J}_{\lambda_2}(k_2)\Big)\gg \ . \tag{13}$$

All terms in (13) involve four space–time integrations. In coordinate space a connected correlator disappears if the distance between any pair of its space–time points exceeds the correlation length. The connected four–point correlator $\ll\cdots\gg$ (last term on the right–hand side of (13)) is only non–zero if the relative distance of all its space–time points are within λ_s . Therefore, this term is proportional to $L^4(\lambda_s^4)^3$ and hence, due to condition (ii) , is smaller by factor of $(\lambda_s/L)^4$ when compared to all other terms which are proportional to $(L^4)^2(\lambda_s^4)^2$. The third term on the r.h.s of (13), $|<\mathcal{T}\Big(\hat{J}_{\lambda_1}(k_1)\hat{J}_{\lambda_2}(k_2)\Big)>|^2$, contains an integral $\int d^4\bar{x}\ \exp(i(|\boldsymbol{k}_1|+|\boldsymbol{k}_2|)\bar{x}^0)\ \cdots$ in which the energies of the two photons add up in the phase factor. This implies that only momenta which fulfill the condition $(|\boldsymbol{k}_1|+|\boldsymbol{k}_2|)\cdot L \lesssim 1$ contribute appreciably to this term. These momenta are much smaller than required by conditions (ii) and (iii) and hence we shall drop this term.

After these approximations based on the statistical properties of the emitting source the one– and two–photon spectra (9) and (10) are given by the current–current correlator $<\hat{J}^{\mu_1\dagger}(k_1)\hat{J}^{\mu_2}(k_2)>$ only. Because the electrical current is conserved $(\partial_\mu\hat{J}^\mu(x)=0)$ the correlator must be transverse "from both sides":

$$k_{1\mu_1}\ <\hat{J}^{\mu_1\dagger}(k_1)\hat{J}^{\mu_2}(k_2)> = <\hat{J}^{\mu_1\dagger}(k_1)\hat{J}^{\mu_2}(k_2)>\ k_{2\mu_2} = 0 \ . \tag{14}$$

In analogy to (11) the correlator can be written as

$$<\hat{J}^{\mu_1\dagger}(k_1)\hat{J}^{\mu_2}(k_2)> = \sum_n \mathrm{e}^{-i(k_1-k_2)\bar{x}_n}\int\limits_{\Omega_n} d^4\bar{x}\,\mathrm{e}^{-i(k_1-k_2)(\bar{x}-\bar{x}_n)}\int d^4\Delta x\,\mathrm{e}^{-i\Delta x(k_1+k_2)/2}$$

$$\times Tr\Big\{\rho_i\hat{J}^{\mu_1}(\bar{x}+\frac{\Delta x}{2})\hat{J}^{\mu_2}(\bar{x}-\frac{\Delta x}{2})\Big\} \ , \tag{15}$$

where the whole space–time region occupied by the source is divided into cells with 4–volume Ω_n located at mean–coordinates $\bar{x}_n$. Under the assumptions $(i)-(iii)$ the size of the space-time cells should be chosen to be about λ_s so that the radiation of hard photons from different cells can be considered as entirely independent processes. This implies that the currents are conserved for each cell separately and that the contribution of the cells to the total current correlator is additive. Therefore, we parametrize (15) as

$$<\hat{J}^{\mu_1\dagger}(k_1)\hat{J}^{\mu_2}(k_2)> \overset{ansatz}{=\!=} \sum_n \Omega_n \mathrm{e}^{-i\bar{x}_n(k_1-k_2)}Q^{\mu_1\mu_2}(k_1,k_2|\bar{x}_n)w(k_1,k_2|\bar{x}_n) \ , \tag{16}$$

where the tensor $Q^{\mu_1\mu_2}$ carries the tensorial structure of the current–current correlator and the function w describes the strength. The Q–tensor is explicitly transverse

$$k_{1\mu_1} Q^{\mu_1\mu_2}(k_1,k_2|x) = Q^{\mu_1\mu_2}(k_1,k_2|x) k_{2\mu_2} = 0 \tag{17}$$

and, therefore, ensures current conservations for every cell. The tensor $Q^{\mu_1\mu_2}$ and the function w are hermitian in the sense that $Q^{\mu_1\mu_2}(k_1,k_2|x)^* = Q^{\mu_2\mu_1}(k_2,k_1|x)$ and $w(k_1,k_2|x)^* = w(k_2,k_1|x)$. It is convenient to normalize the Q–tensor by $Q^\mu_\mu(k,k|x) \equiv -2$. From now on we will write an integral instead of the sum in (16).

Altogether single and double inclusive photon spectra (no polarization measured) can be expressed with the help of (16) as follows[1]:

$$P_1(\boldsymbol{k}) \equiv \sum_{(\lambda=1)}^{2} P_1^\lambda(\boldsymbol{k}) = 2\int d^4x\, w(k,k|x) \tag{18}$$

$$P_2(\boldsymbol{k}_1,\boldsymbol{k}_2) \equiv \sum_{(\lambda_1,\lambda_2=1)}^{2} P_2^{\lambda_1\lambda_2}(\boldsymbol{k}_1,\boldsymbol{k}_2) = P_1(\boldsymbol{k}_1)P_1(\boldsymbol{k}_2) \tag{19}$$

$$+\int d^4x d^4y\, \mathrm{e}^{-i(x-y)(k_1-k_2)} w(k_1,k_2|x) w(k_2,k_1|y) Q^{\mu_1\mu_2}(k_1,k_2|x) Q_{\mu_2\mu_1}(k_2,k_1|y)$$

To find an explicit form of the Q–tensor we assume that the radiating system is in local equilibrium, that is, the radiation from an elementary cell is isotropic in its local rest frame and therefore $Q^{i_1 i_2} = \delta^{i_1 i_2}$ (where $i_1, i_2 = 1,2,3$). The other components of the Q–tensor are then uniquely determined by the transversality condition (17) as $Q^{00} = \boldsymbol{k}_1\boldsymbol{k}_2/k_1^0 k_2^0$, $Q^{i_1 0} = k_2^{i_1}/k_2^0$, $Q^{0 i_2} = k_1^{i_2}/k_1^0$. In local thermal equilibrium the only other 4–vector entering is the 4–velocity $u^\mu(x)$ of the emitting cell which is located at point x. Therefore, the only hermitian and transversal Q–tensor which can be constructed from k_1^μ, k_2^μ and $u^\mu(x)$, and which in the rest frame of a cell ($u^\mu = (1,\mathbf{0})$) reproduces the isotropic form given above, reads as follows:

$$Q^{\mu_1\mu_2}(k_1,k_2|x) = -g^{\mu_1\mu_2} - u^{\mu_1}(x)u^{\mu_2}(x)\frac{(k_1k_2)}{(k_1u(x))(k_2u(x))} + \frac{u^{\mu_1}(x)k_1^{\mu_2}}{(u(x)k_1)} + \frac{k_2^{\mu_1}u^{\mu_2}(x)}{(u(x)k_2)}\ . \tag{20}$$

In eq. (20) we do not include the term proportional to $k_1^{\mu_1}k_2^{\mu_2}$ because it does not contribute to observables and can be removed by an appropriate gauge transformation.

[1] We take the sum over photon polarisations by means of

$$\sum_{(\lambda=1)}^{2} \overset{*}{\epsilon}{}^\lambda_\mu(k)\epsilon^\lambda_\nu(k) = -g_{\mu\nu} - k_\mu k_\nu/(sk)^2 + (k_\mu s_\nu + s_\mu k_\nu)/(sk)$$

(s^μ is some reference 4-vector $s^2 = 1$), where due to the transversality condition (17) only the first term on the right–hand side $(-g_{\mu\nu})$ really contributes to (18), (19).

After defining the Q–tensor all dynamic information on the photon production is contained in the function $w(k_1, k_2|x)$. Inserting (20) into (19) one gets

$$P_2(\boldsymbol{k}_1, \boldsymbol{k}_2) = P_1(\boldsymbol{k}_1)P_1(\boldsymbol{k}_2) + \int d^4x d^4y \cos(\Delta x \Delta k) R(k_1, k_2|x, y) w(k_1, k_2|x) w(k_2, k_1|y) \ , \quad (21)$$

where we introduce the abbreviation

$$R(k_1, k_2|x, y) \equiv Q^{\mu_1\mu_2}(k_1, k_2|x) Q^{\mu_2\mu_1}(k_2, k_1|y) = \frac{(u(x)k_1)(u(y)k_2) - 2(u(x)u(y))(k_1k_2)}{(u(x)k_2)(u(y)k_1)}$$

$$+ \frac{(k_1k_2)}{(k_1u(x))(u(x)k_2)} + \frac{1}{2}\frac{(u(x)u(y))^2(k_1k_2)^2}{(k_1u(x))(u(x)k_2)(k_1u(y))(u(y)k_2)} + \Big(x \Leftrightarrow y\Big) \ . \quad (22)$$

The R–function reflects the fact that photons are massless spin–1 particles and in this point expression (21) differs significantly from the one obtained in [5] for pions. The latter has been used for photons of one single polarization in [2, 3, 9, 10]. Since the experimental measurements will involve an averaging over polarizations, a comparison with data requires the new result (21). Being very sensitive to the 4–velocity field $u^\mu(x)$ the R–function can change drastically the interference of two photons for a relativistically expanding source. But even for a static source the polarization properties of photons are very important. To illustrate this let us consider two photons with momentum $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ which are emitted from two cells having the same 4–velocity $u(x) = u(y) = (1, \boldsymbol{0})$. In that case [1] the R–function reduces to

$$R(k_1, k_2|x, y) = 1 + (\cos\theta)^2 = \sum_{(\lambda_1, \lambda_2 = 1)}^{2} \Big(\boldsymbol{\epsilon}^{\lambda_1}(k_1)\boldsymbol{\epsilon}^{\lambda_2}(k_2)\Big)^2 \ , \quad (23)$$

where $\cos\theta = \boldsymbol{k}_1\boldsymbol{k}_2/(|\boldsymbol{k}_1||\boldsymbol{k}_2|)$. The physical reason is easily understood for this special case when one considers the last part of equation (23). The summation over polarizations (here we choose the Coulomb gauge: $\epsilon^\lambda(k) = (0, \boldsymbol{\epsilon}^\lambda(k))$ and $\boldsymbol{k}\boldsymbol{\epsilon}^\lambda(k) = 0$) does not just lead to a factor of two as implicitly assumed in [2, 3, 9, 10]. As illustrated by Fig. 1 only one direction of the linear polarization can be chosen equal for both photons, whereas the other polarization directions differ by the angle θ between the two momenta. Therefore, the polarization overlap involves $(\cos\theta)^2$ and is less than 2 for $\theta \neq 0$ (see (23)). In a realistic situation the radiation comes of course from many cells with different four–velocities and the full structure of the Q–tensor has to be employed. Anyhow, for a non–vanishing angle between the observed photon momenta the correlation is reduced compared to the "pion inspired" recipe (25).

For application in dynamical models we recommend to use the following formula:

$$P_2(\boldsymbol{k}_1, \boldsymbol{k}_2) = P_1(\boldsymbol{k}_1)P_1(\boldsymbol{k}_2) + <\hat{J}^{\mu_1\dagger}(k_1)\hat{J}^{\mu_2}(k_2)><\hat{J}^{\dagger}_{\mu_2}(k_2)\hat{J}_{\mu_1}(k_1)> \quad (24)$$

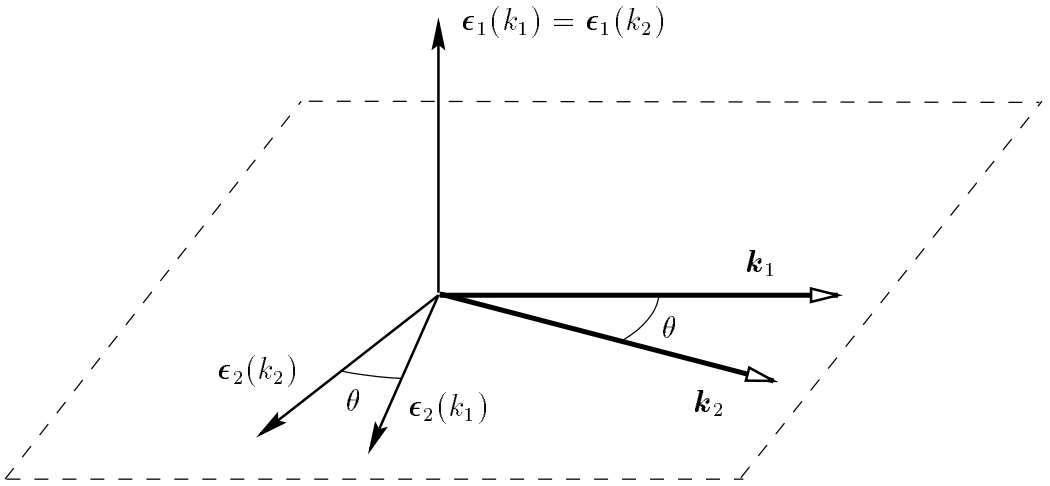

Fig. 1: The linear polarization vectors $\epsilon^\lambda(k)$ for two photons with momenta $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$.

and calculate the current–current correlator with (16) keeping the explicit form of the Q–tensor as given in (20). This is equivalent to (21) but involves only one d^4x integration instead of $d^4x\, d^4y$ in (21).

The dependence of $w(k_1, k_2|x)$ on $\Delta k = k_1 - k_2$ and $\bar{k} = (k_1 + k_2)/2$ has to be derived from a microscopic model for the photon production. The only general statement which can be made at this point is that due to condition (i) the current–current correlator decays in the local restframe as function of Δk on the long scale λ_s^{-1}. In this paper we do not consider a specific microscopic model for $w(k_1, k_2|x)$ but rather investigate the role of the tensorial structure of the correlator on photon–photon interferometry. Therefore we write $w(k_1, k_2|x) = w(\bar{k}, \bar{k}|x) + \mathcal{O}(\Delta k^\mu \Delta k^\nu)$. For simplicity we drop terms of second order in Δk. The strength of photon production $w(\bar{k}, \bar{k}|x)$ can to some extend be estimated from the single inclusive cross section [11].

Based on the discussion above we propose to reexamine the photon intensity interferometry in the spirit of [2, 3, 9, 10] but using (24) for the double inclusive cross section.

Unfortunately until now the Bose–Einstein correlations of photons have been studied using the formula for double inclusive spectra where the photons are assumed to be scalar massless particles, but normalizing the correlation function to 3/2 instead of 2 [9, 2, 10]. That formula is just expression (21) with $R(k_1, k_2|x, y) = 2$ and reads

$$P_2^{wrong}(\boldsymbol{k}_1, \boldsymbol{k}_2) = P_1(\boldsymbol{k}_1) P_1(\boldsymbol{k}_2) + 2 \int d^4x d^4y \, \cos(\Delta x \Delta k) w(\bar{k}, \bar{k}|x) w(\bar{k}, \bar{k}|y) \quad . \tag{25}$$

In order to demonstrate the size of the effect which arises from the vector nature of the photon and the conservation of electric charge we have performed numerical studies of photon intensity interferometry in Bjørken hydrodynamics (equation of state $p = \epsilon/3$)

using both our formula (24) (or equivalently (21)) and the wrong one (25). We assume that the photons are produced from the expanding source in local equilibrium and parameterize the photon production rate as $w(k_1, k_2|x) \cong w(\bar{k}, \bar{k}|x) = N \cdot (T(x))^2 \exp(-(\bar{k}u(x))/T(x))$ (for a more precise expression cf. [11]). The plot of the correlation function defined as $C_2(k_1, k_2) \equiv P_2(k_1, k_2)/P_1(k_1)P_1(k_2)$ shows a significant difference in the results based on the different formulas (see Fig. 2). The correct expression gives less correlations for non–zero relative rapidities which means that the source size deduced from experimental data is overestimated by the wrong formula (25).

In summary we should like to stress that the derivation of a basic equation for the double inclusive spectrum is significantly modified by the fact that photons are massless particles with spin 1 and that they are produced by a conserved electric current. Under the conditions $(i) - (iii)$ which are fulfilled for high–energy photons in relativistic heavy–ion collisions, the expression (24) is a suitable starting point for photon intensity interferometry studies.

We would like to acknowledge the fruitful discussions with G. Bertsch and J. Knoll as well as the comments of D. Seibert. Especially we are grateful to R. Weiner and M. Plümer for critical remarks.

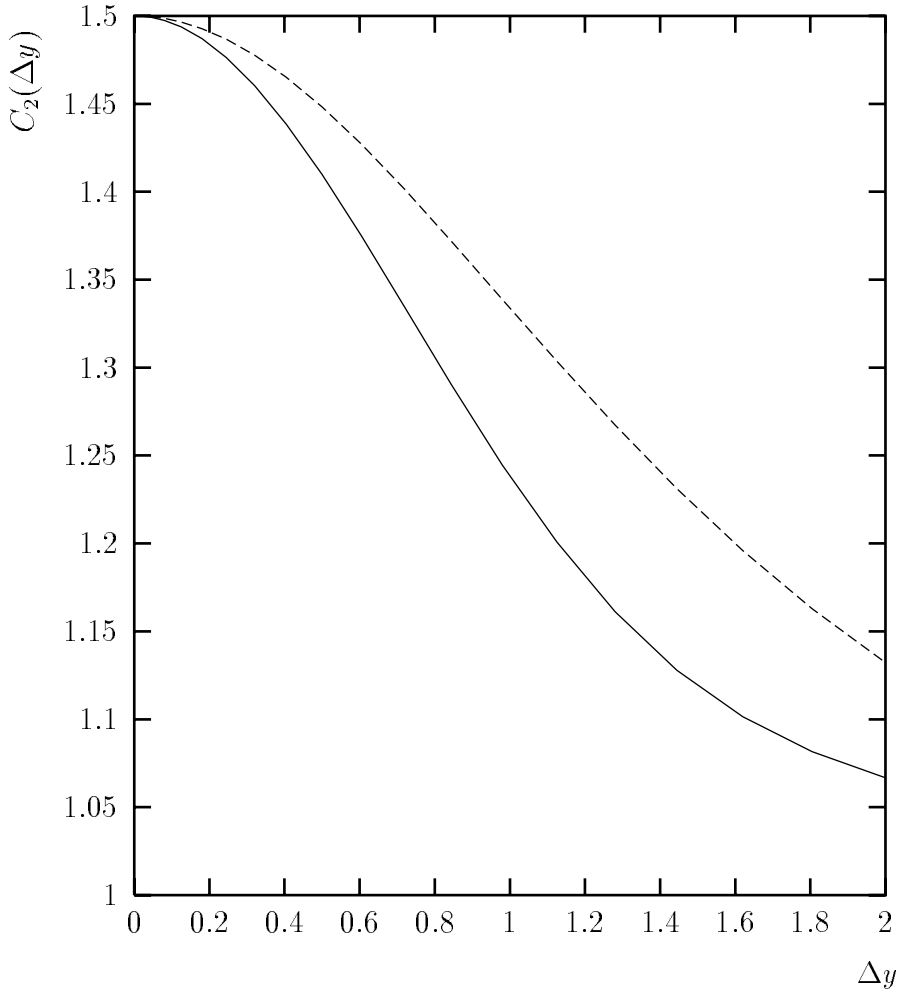

Fig. 2: Bose–Einstein correlation plot as a function of rapidity difference Δy at fixed transverse momenta $\mathbf{k}_1^\perp = \mathbf{k}_2^\perp = 100$ MeV for Bjørken hydrodynamics with $T_i = 200$ MeV, $T_f = 140$ MeV and initial proper–time $\tau_i = 0.3\ fm/c$.
Solid line corresponds to the photon–photon correlations (24) (or equivalently (21)).
Dashed line represents the wrong "pion–inspired" expression (25).

References

[1] D. Neuhauser, Phys. Lett. **B182** (1986) 289.

[2] D. K. Srivastava and J. I. Kapusta, Phys. Rev. **C 48** (1993) 1335.

[3] A. Timmermann, M. Plümer, L. V. Razumov, and R. M. Weiner, Phys. Rev. **C 50** (1994) 3060.

[4] M. Marques et al., Phys. Rev. Lett. **73** (1994) 34.

[5] S. Pratt, Phys. Rev. Lett. **53** (1984) 1219.

[6] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields*, McGraw-Hill, 1965.

[7] A. Makhlin, Sov.J.Nucl.Phys. **49** (1989) 238.

[8] Leonid. V. Razumov and R. M. Weiner, Phys. Lett. **B 348** (1995) 133.

[9] D. K. Srivastava and J. I. Kapusta, Phys. Lett. **B 307** (1993) 1.

[10] D. K. Srivastava and J. I. Kapusta, Phys. Rev. **C 50** (1994) 505.

[11] J. Kapusta, P. Lichard, and D. Seibert, Nucl. Phys. **A 544** (1992) 485c.